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13. ABSTRACT (Maximum 200 words)

Brief summaries are given of research performed in the following areas: Adaptive Euler equation solvers; adaptation parameters for vortical flow; vortex breakdown calculations; calculations for F117A; normal force hysteresis; visualization of vortical flows on unstructured grids; and modeling of vortex breakdown. The reference list gives reports with detailed results.

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Adaptive Navier-Stokes Calculations for Vortical Flows

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Air Force Office of Scientific Research

Dr. Leonidas Sakell
AFOSR/NA Building 410
Bolling Air Force Base
Washington, DC 20332
(202) 767-4987
FAX(202) 767-0466

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Submitted by
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology
Cambridge, MA 02139

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Grants Officer
Charlotte Morse
(617) 253-3258
FAX (617) 253-8000

Principal Investigator
Professor Earl M. Murman
(617) 253-3284
FAX (617) 258-7566

INTRODUCTION

This is the final report of a multi-year research project for modeling vortical flows with adaptive Navier-Stokes algorithms. The objectives of the research were to:

Develop a three-dimensional adaptive Navier-Stokes computational model,

Investigate adaptation criteria and turbulence models suitable for vortical flows, and

Compute delta wing and/or canard-wing flows, analyze the results, and compare with data.

Considerable progress was made in understanding the physics of vortical flows through three-dimensional calculations, interactive flow visualization, and theoretical modeling. Of particular interest was vortex bursting, both steady and non-steady. An adaptive Euler equation solver was developed and utilized to calculate flows for simple delta wing geometries as well as an F117A configuration. The objective of developing adaptive Navier-Stokes capability was not achieved. This was partly due to the fact that it took longer than we anticipated to develop the Euler part of the solver, and partly to the fact that vortex bursting is sensitive to viscous effects for sharp leading edge geometries. During the grant period the Principal Investigator became aware of the interesting and important aerodynamic problem of normal force hysteresis for rapidly pitching wings and aircraft. The hysteresis is caused by a lag in the vortex bursting position. Computational studies were undertaken to understand this problem, even though unsteady flows were not in the original list of objectives.

A number of publications emerged from the research, and all are included in the list of references. In the following pages, the most important findings from this work are summarized. The reader is referred to the individual publications for further details.

RESEARCH RESULTS

Adaptive Euler Equation Solver

A considerable portion of the three year grant was focused on the development of a code for an adaptive unstructured (tetrahedra) grid solver for the Euler equations. The underlying algorithms were developed and implemented from first principles. A trilinear Galerkin Finite Element Method was used for spatial discretization and the Jameson four stage Runge-Kutta type scheme was used for temporal integration. A blended second-fourth difference artificial viscosity based upon the Holmes-Connell-Saxer approach was adopted. Boundary conditions for far field, solid body, symmetry plane and sharp edges were implemented. This explicit method was run using local time steps for steady state problems. For unsteady problems, a novel acceleration factor was introduced which provided an order of magnitude reduction in computing time for a three-dimensional problem (Reference 11). A global time step was chosen and used for all cells where the minimum time step exceeded the global value. For the other cells, the local time step was used. Although this approach sacrifices locally accuracy in some cells, *a priori* and computational experiments established acceptable levels of global accuracy. The method is very robust and extremely easy to implement. It is ideal for parallel applications avoiding the difficulties of block factored implicit methods. A full account of the solver is given in Reference 8¹ and an abbreviated account is given in Reference 12.

Adaptation parameters for vortical flows

In results reported in Reference 2, we experimented with different criteria for adaptive calculations of vortical flows. Most adaptive calculations we are aware of have been designed to detect and refine shock waves. One cannot expect the same feature finders used for shock waves will work with vortices. We explored two parameters for adaptation. One was the total pressure loss which is well known to occur in vortical flow. The other was normalized helicity, the dot product of vorticity and velocity normalized by the absolute values of the two quantities. This is just the cosine of the angle between the velocity and vorticity vector which should be unity in the core of a vortex. A full discussion of the adaptation methods is given in Landsberg's thesis, Reference 2. The key results are contained in a review paper written by Landsberg and Murman (Reference

¹ A listing of the code is available upon request.

7) which discusses various methods, including adaptation, for controlling the numerical diffusion of vortices.

In the extension to unsteady flows reported in References 8 and 12, entropy was used instead of total pressure. Entropy is a state variable independent of the reference frame. Our conclusion was that entropy is easy to implement and a suitable parameter for finding vortices, at least for the Mach number range considered in this study.

Vortex Breakdown Calculations

The adaptive Euler Solver summarized above was used to compute the flow about a delta wing with 75 degree leading edge sweep, a Mach number of 0.3, and for angles of attack from 0 to 52 degrees. The base non-adapted grid had about 15,000 nodes while the adapted grid had about 80,000 nodes. Normal force results agree well with experimental data through the entire range of angles of attack when adaptation is used. Without adaptation, the grid resolution is insufficient to capture vortex breakdown. Results for pitching moment and vortex breakdown position are in reasonable agreement with the data. Computations with breakdown present do not reach a steady state solution, most probably due to the unsteady nature of the burst vortex. Small oscillations in forces and bursting position persist. However, the solutions are steady in the large. Comparisons at several angles of attack are made with published results using a much finer non adaptive structured grid and agreement is satisfactory. Detailed analysis of the flow field reveal interesting features of the breakdown. At 32 degrees angle of attack, a spiral breakdown is predicted. At 42 degrees angle of attack, a bubble breakdown followed by reformation of the vortex and a spiral breakdown is predicted. Detailed results are given in References 8 and 12.

Calculations for the F117A

The adaptive Euler solver was applied to the F117A (Reference 13) configuration to test its ability to handle arbitrary geometry. A plastic model was digitized for the geometrical definition. Results were obtained at 15, 20, and 25 degrees angle of attack. Vortex bursting was absent in the first case. For 20 degrees angle of attack, bursting was about at the trailing edge. Bursting was well over the wing for the third case. Subsequent wind tunnel studies have shown general agreement with these findings.

Normal Force Hysteresis

During the grant period, we were made aware of the interesting experimental results which show significant normal force hysteresis for pitching delta wings and for a Russian aircraft. Dynamic force coefficients can exceed static values by 30-50% on the upstroke portion of the pitch, and can undershoot static values by lesser, but still significant values on the down stroke portion. Experimental evidence shows this is due to a lag in the vortex bursting position, thereby maintaining the augmented normal forces to a higher angle of attack on the upstroke. A lag in the opposite direction on the down stroke causes the undershoot. A computational experiment was done to determine if this effect could be predicted by the adaptive Euler solver. Preliminary results were obtained (Reference 12) showing the general effect. Both time and resources ran out before more detailed calculations could be done.

Visualization of vortical flows on unstructured grids

Visualization of the complex vortical flows proved to be an interesting and important challenge in order to understand the results (References 1, 3, 4, 5, 6). Considerable attention was paid to developing novel visualization strategies and evaluating their usefulness. The foundation for the visualization work is the VISUAL 3 software developed by Giles and Haines in the MIT Computational Fluid Dynamics Laboratory. This software treats generic three-dimensional data sets on unstructured grids, and it runs interactively on high end graphics workstations. To VISUAL 3 we added a number of capabilities which provide for a hierarchy of approaches for analyzing complex flow fields. The strategy is to start with *Identification* methods which locate the major regions of interest in the flow field. Several new identification methods were developed including X-rays, vector clouds, and a shock finder. When the important regions are found, the next step is to use *Scanning* methods to zero in on detailed features. Cutting planes and iso surfaces are examples of these. The final step is to *extract* quantitative data with *Probing* techniques. A variety of interactive **probes** were developed, and significantly improved particle path methods were introduced. The latter permit viewing not only the path tangent to a vector field, but also its divergence and curl to be seen. Methods for determining grid quality were also investigated (Ref 9).

Modeling of Vortex Breakdown

In order to better understand the ability of inviscid computational methods to capture vortex breakdown, studies were undertaken to

compute and analyze an isolated axisymmetric vortex subjected to a controlled adverse pressure gradient. An incompressible Navier-Stokes solver was written and applied to a vortex in a tube with a converging-diverging nozzle. The breakdown location can be controlled by the inlet swirl, the Reynolds number, and the divergence angle of the nozzle. Of these parameters, the Reynolds number effects are insignificant above relatively low values.

Theoretical studies have also been done from the point of view of vorticity dynamics. It was found the important effect leading to vortex breakdown is the tilting of the axial vorticity vector into the azimuthal direction as the streamtubes expand radially due to the adverse pressure gradient. This is an inviscid effect which is in agreement with the observation that Reynolds number plays an insignificant role. These results will be presented at an upcoming AIAA meeting (Ref 10).

Turbulence modeling

For Reynolds numbers applicable to flight, leading edge vortices have turbulent cores. Yet little work has been done to develop turbulence models for use in CFD studies of vortices. Most investigators who solve the Reynolds Averaged Navier-Stokes equations use turbulence models developed for wall bounded flows. There is no reason to believe these are suitable for vortical flows. Preliminary work was done to develop a turbulence model for vortex flows. The results are given in the Appendix. However, when the goal of developing a Navier-Stokes solver was abandoned, this task became of lesser importance and it was abandoned.

Parallel processing study

We expended some effort on mapping the above unstructured Euler program onto the Kendall Square Research parallel computer prior to the date it was available. Our studies showed that the unstructured grid program would achieve about 6 gigaflops on the full 1K node processor. Although we expected to implement the results on the KSR machine, delays in its availability prevented us completing this task.

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PROFESSIONAL PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT

Principal Investigator

Professor Earll M. Murman

Other faculty

Professor Eugene Covert
Professor Mårten Landahl

Graduate Students

Alexandra Landsberg	Research Assistant, SM November 1990
David Darmofal	DOD Fellow, SM June 1991, PhD Candidate
David Modiano	Research Assistant, PhD January 1993
Sabine Vermeersch	Research Assistant, SM expected June 1993

Professional Staff

Steve Ellis **Research Engineer**

INTERACTIONS

Seminar Presented at Flight Dynamics Laboratory, Wright Aeronautical Laboratories on February 2, 1990

Seminar presented at the Arnold Engineering Development Center on August 5, 1992.

Invited Presentation on "Computational Challenges for Vortical Flows" at the US Air Force International CFD Symposium for Air Vehicle Technology at Wright Laboratory, Sept 14-16, 1992.

NEW DISCOVERIES

None.

APPENDIX

TURBULENCE MODEL FOR A ROTATING FLOW

M.T. Landahl

1. Introduction

The modelling of the turbulent stresses in a flow with strong rotation is an important problem in the calculation of flow in rotating machinery and in strong vortices. How to include the effect of rotation in turbulence models like the k - ϵ -model has been discussed by many, see Speciale (1991). A recent treatment of the problem by Ekander and Johansson (1989) has led to a simple algebraic Reynolds stress model (ARSM) which appears to include the effect of rotation in a consistent manner.

2. Model fundamentals

We will consider an incompressible turbulent flow in a Cartesian frame of reference rotating steadily around the z -axis by the angular velocity ω . We subdivide the velocity components and pressure in mean and fluctuating components by setting

$$U_i(x_j, t) = \bar{U}_i + u_i(x_j, t), \quad P = \bar{P} + p(x_j, t), \quad \langle u_i \rangle = 0, \quad \langle p \rangle = 0 \quad (1)$$

The momentum equations for the mean quantities (assumed steady) read

$$\bar{U}_j \frac{\partial \bar{U}_i}{\partial x_j} + 2\omega \epsilon_{ijk} \bar{U}_k = -\frac{1}{r} \left[\frac{\partial \bar{P}^*}{\partial x_i} - \frac{\partial}{\partial x_j} \left(m \frac{\partial \bar{U}_i}{\partial x_j} + \bar{t}_{ij} \right) \right] \quad (2)$$

where the Reynolds stresses \bar{t}_{ij} are defined by

$$\bar{t}_{ij} = -r \langle u_i u_j \rangle \quad (3)$$

and \bar{P}^* is the reduced pressure including the centrifugal effect. The fluctuating components satisfy

$$\frac{\bar{D} u_i}{\bar{D} t} + u_j \frac{\partial \bar{U}_i}{\partial x_j} + 2\omega \epsilon_{ijk} u_k = -\frac{1}{r} \left[\frac{\partial p^*}{\partial x_i} - \frac{\partial}{\partial x_j} \left(m \frac{\partial u_i}{\partial x_j} + t_{ij} \right) \right] \quad (4)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (5)$$

where

$$\frac{\bar{D}}{\bar{D} t} = \frac{\partial}{\partial t} + \bar{U}_j \frac{\partial}{\partial x_j} \quad (6)$$

and where

$$t_{ij} = -r(u_i u_j - \langle u_i u_j \rangle) \quad (7)$$

For the two-dimensional case with rotation around the z-axis the equations for the fluctuating components read, with $u_1 = u$, $u_2 = v$, $u_3 = w$

$$\frac{\bar{D}u}{Dt} + u \frac{\partial \bar{U}}{\partial x} + v \frac{\partial \bar{U}}{\partial y} - 2wv = -\frac{1}{r} \left[\frac{\partial p^*}{\partial x} - \frac{\partial}{\partial x_j} \left(m \frac{\partial u}{\partial x_j} + t_{ij} \right) \right] \quad (8)$$

$$\frac{\bar{D}v}{Dt} + u \frac{\partial \bar{V}}{\partial x} + v \frac{\partial \bar{V}}{\partial y} + 2wu = -\frac{1}{r} \left[\frac{\partial p^*}{\partial y} - \frac{\partial}{\partial x_j} \left(m \frac{\partial v}{\partial x_j} + t_{ij} \right) \right] \quad (9)$$

$$\frac{\bar{D}w}{Dt} = -\frac{1}{r} \left[\frac{\partial p^*}{\partial z} - \frac{\partial}{\partial x_j} \left(m \frac{\partial w}{\partial x_j} + t_{ij} \right) \right] \quad (10)$$

The equations for the Reynolds stresses of interest, $\bar{t}_{11} = -r\langle u^2 \rangle$, $\bar{t}_{12} = -r\langle uv \rangle$ and $\bar{t}_{22} = -r\langle v^2 \rangle$ are obtained by multiplying the equation for u by u , then by v , and that for v by u , then by v and averaging and adding. After using the equation of continuity (5) one obtains transport equations for the Reynolds stresses which may be written, following Ekander and Johansson (1989) as follows:

$$\frac{\bar{D}' \bar{t}_{ij}}{\bar{D}'t} - D_{ij} = P_{ij} + P_{ij} + \theta_{ij} \quad (11)$$

where $\frac{\bar{D}' \bar{t}_{ij}}{\bar{D}'t}$ is the "true" time rate of change defined by

$$\frac{\bar{D}' \bar{t}_{ij}}{\bar{D}'t} = \frac{\bar{D} \bar{t}_{ij}}{Dt} + E_{ij} \quad (12)$$

with E_{ij} being the redistribution of the Reynolds stresses due to the rotation, given in the two-dimensional case by

$$E_{11} = -2w \langle uv \rangle, E_{12} = w (\langle u^2 \rangle - \langle v^2 \rangle), E_{22} = 2w \langle uv \rangle \quad (13)$$

D_{ij} is the diffusive rate of change defined by

$$D_{ij} = \frac{\partial \langle u_k u_i u_j \rangle}{\partial x_k} + \frac{1}{r} \frac{\partial \langle u_i p \rangle}{\partial x_j} + \frac{1}{r} \frac{\partial \langle u_j p \rangle}{\partial x_i} \quad (14)$$

P_{ij} the production

$$P_{ij} = -\bar{t}_{ij} \frac{\partial \bar{U}_i}{\partial x_j} + \bar{t}_{ij} \frac{\partial \bar{U}_j}{\partial x_i} - w \epsilon_{ijk} \bar{t}_{ik} - w \epsilon_{ijk} \bar{t}_{jk} \quad (15)$$

P_{ij} the pressure-strain correlation

$$P_{ij} = \frac{1}{r} \langle p \frac{\partial u_i}{\partial x_j} \rangle + \frac{1}{r} \langle p \frac{\partial u_j}{\partial x_i} \rangle \quad (16)$$

and θ_{ij} the viscous dissipation,

$$\theta_{ij} = -n \left\langle \frac{\partial u_i}{\partial x_k} \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \right\rangle - n \left\langle \frac{\partial u_j}{\partial x_k} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \right\rangle \quad (17)$$

It should be noted that one half of the contribution resulting from the Coriolis term has been kept on the left-hand side of the evolution equation (8) on the argument that it should be considered part of the substantial rate of change. This results from the requirement that the evolution equations should be invariant under arbitrary Galilean transformations. A problem with earlier treatments of the problem has been that all of this term has been incorporated incorrectly into the production term.

By taking the trace of the Reynolds stresses one obtains the usual equation for the kinetic energy: with $k = (1/2)\langle u_i u_i \rangle$

$$\frac{\bar{D}k}{Dt} = (1/2r) \frac{\bar{D}t_{ii}}{Dt} = (1/2)(P_{ii} + P_{ii} - D_{ii}) - \theta \quad (18)$$

where θ is the dissipation rate

$$\theta = 2n \left\langle \left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_i}{\partial x_i} \right)^2 \right\rangle \quad (19)$$

For the two-dimensional case considered one thus finds (note that $E_{ij}=0$)

$$\frac{\bar{D}k}{Dt} = -\langle uv \rangle \frac{\partial \bar{U}}{\partial y} - \theta \quad (20)$$

For the individual production terms P_{ij} we have in this case

$$P_{11} = -2\langle uv \rangle \frac{\partial \bar{U}}{\partial y} - 2W\langle uv \rangle, P_{12} = -\langle v^2 \rangle \frac{\partial \bar{U}}{\partial y} + W(\langle u^2 \rangle - \langle v^2 \rangle), P_{22} = 2W\langle uv \rangle \quad (21)$$

In the case of a curvilinear coordinate system the derivatives of the base vectors will enter in the redistribution terms E_{ij} . For a circular channel with rotation about the origin one finds with $u_1 = u_r$

, $u_2 = u_\theta$, and $\bar{t}_{11} = -r\langle u_r u_r \rangle$, $\bar{t}_{12} = -r\langle u_r u_\theta \rangle$, $\bar{t}_{22} = -r\langle u_\theta u_\theta \rangle$

$$P_{11} = 2\langle u_r u_\theta \rangle \frac{\bar{U}_\theta}{r}, P_{12} = -2\langle u_r u_r \rangle \frac{\partial \bar{U}_\theta}{\partial r} + 2\langle u_\theta u_\theta \rangle \frac{\bar{U}_\theta}{r}, P_{22} = -2\langle u_r u_\theta \rangle \frac{\partial \bar{U}_\theta}{\partial r} \quad (22)$$

$$E_{11} = 2\langle u_r u_\theta \rangle \frac{\bar{U}_\theta}{r}, E_{12} = (\langle u_\theta u_\theta \rangle - \langle u_r u_r \rangle) \frac{\bar{U}_\theta}{r}, E_{22} = -2\langle u_r u_\theta \rangle \frac{\bar{U}_\theta}{r} \quad (23)$$

3. Construction of an algebraic stress model

The fundamental idea is to construct algebraic relationships between the individual components \bar{t}_{ij} and the kinetic energy k and the dissipation rate θ . To this purpose an approximation suggested by Rodi (1976) was utilized in which he suggested that the difference between the advective and diffusive terms of the individual components \bar{t}_{ij} , divided by the corresponding difference in the equation for k , should be set equal to the ratio between the respective Reynolds stress component and k . This then leads to the relation

$$\frac{\frac{D\bar{t}_{ij}}{Dt} - D_{ij}}{\frac{Dk}{Dt} - D_{kk}} = \frac{\bar{t}_{ij}}{k} \quad (24)$$

In this manner the problem of determining the Reynolds stresses is simplified to that of solving the evolution equations for k and θ . For these one needs modelling of all the terms, except the production and the redistribution terms.

The 'slow part' of the pressure strain term (i.e., the part involving nonlinear fluctuation terms in the pressure equation) is modeled, following Rotta (1951) as proportional to the anisotropy of the Reynolds stress tensor :

$$P_{ijs} = -c_s \frac{\theta}{k} (\bar{t}_{ij} - \frac{2}{3} k \delta_{ij}) \quad (25)$$

where $c_s = 1.9$. The rapid part is modeled, following Reynolds (1970), as being proportional to the anisotropy of the production tensor:

$$P_{ijr} = -c_r \frac{\theta}{k} (P_{ij} - \frac{2}{3} P \delta_{ij}) \quad (26)$$

where P is the trace of the production tensor and $c_r = 0.6$. The presence of the wall will also transfer energy from the transverse to the streamwise component through reflections at the wall. In the paper of Ekander and Johansson (1989) this effect is modelled in a simple manner by subtracting from the normal component the value

$$-c_w \frac{\theta}{k} (P_{aa} - \frac{2}{3} P) L \quad (27)$$

and adding this to the streamwise component. In (27), $c_w = 0.72$ and $L = \min(l_0/y, 1.0)$, y denoting the distance to the wall and $l_0 = (c_k k)^{3/2} / (\theta k)$ with $c_k = 0.24$ and $k = 0.41$ (the von Karman constant). Also, the Greek index symbols are used to indicate that summation shall not be applied. The diffusion rate is modelled, following Daly and Harlow (1970), as

$$D = c_d \frac{\partial}{\partial x_i} \left[\left(\frac{\theta}{k} \right) (\bar{t}_{ij} \frac{\partial k}{\partial x_j}) \right], c_d = 0.22 \quad (28)$$

For the dissipation it is usually assumed that the smallest scales, which presumably carry most of the viscous dissipation, are isotropic, hence

$$\theta_{ij} = d_{ij} 2\theta/3 \quad (29)$$

The standard modelled equation for θ is modified to be consistent with the k -equation as follows:

$$\frac{\partial \theta}{\partial t} + \bar{U}_k \frac{\partial \theta}{\partial x_k} - c_\theta \frac{\partial}{\partial x_i} \left[\frac{k}{\theta} \bar{\tau}_{ij} \frac{\partial \theta}{\partial x_j} \right] = -c_{1\theta} \frac{\theta}{k} \bar{\tau}_{ij} \frac{\partial \bar{U}_j}{\partial x_i} - c_{2\theta} \frac{\theta^2}{k} \quad (30)$$

with the following constants chosen: $c_\theta = 0.18$, $c_{1\theta} = 1.44$, $c_{2\theta} = 1.92$.

In their application to channel flows Ekander and Johansson (1989) also had to apply a wall correction for the inner part of the inner inertial layer, $y^+ < 30$. They used a form proposed by Rodi (1980).

Their results for the flow in a nonrotating channel, a rotating two-dimensional channel, and for a circular channel showed good both qualitative and quantitative agreement with experiments. Their results are reproduced in Figs. 1--7.

4. Conclusions

The model proposed by Ekander and Johansson (1989) for turbulence in a rotating frame of reference appears self-consistent and gives results in good agreement with experiments. Their model should be applicable to the flow in a strong vortex.

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